Aalto University
School of Business

Predicting choices with a linear value function in a four-criteria MCDM problem

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Introduction

Subjects can easily make judgments of importance

- *”Rent is more important than size”*

How accurate are these statements?

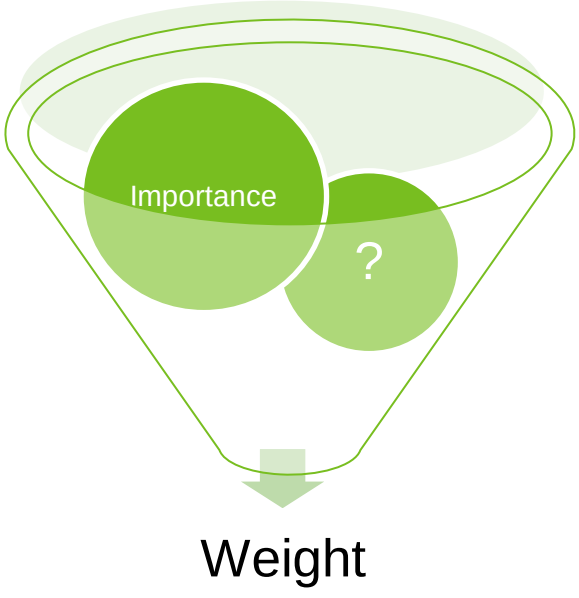
How well do they relate to actual choices?

Research questions

Can we predict choices with a linear value function?

What, if any, is the relation of

- *judgments of importance, and?*
- *weights of the linear value function?*



The Epsilon Model

Linear value function model

$$\textit{Where } X_r \succcurlyeq X_s \textbf{ and } \mathbf{X_r} = [\mathbf{x_{r1}}, \mathbf{x_{r2}}, \mathbf{x_{r3}}, \mathbf{x_{r4}}]$$

Max ϵ subject to:

$$\sum_{j=1}^{4} \lambda_j x_{rj} - \epsilon \geq \sum_{j=1}^{4} \lambda_j x_{sj} \; for \; all \; (X_r, X_s) \in P$$

$$so \; that \sum_{j=1}^{4} \lambda_j = 1$$

- See also Korhonen et al. (2013)

Participants

147 students at Aalto University

Students completed the task on a web-based software

100 randomly chosen full responders were given a movie ticket (worth approx. 9€)

Decision task: Choosing apartments

Demographic data (gender, age, year of study)

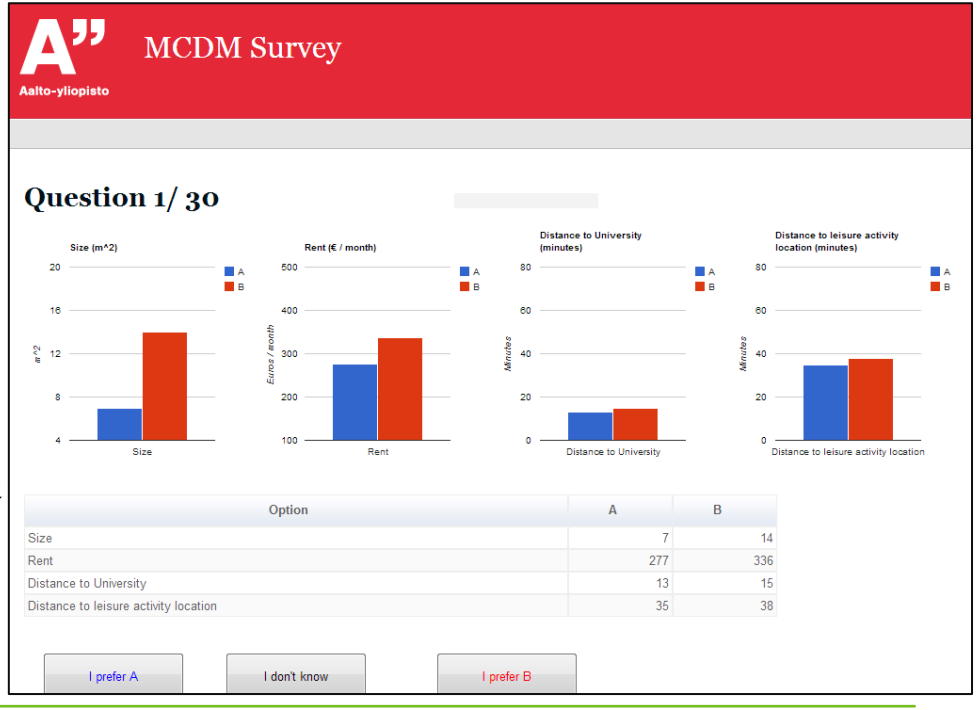
Setting the criteria bounds

- *Size*
- *Rent*
- *Travel time to University*
- *Travel time to leisure location*

AHP-style importances

- *”Rent is greatly more important than size”*

30 Pairwise choice tasks



Decision task: Choosing apartments

30 Pairwise choice tasks

Questions	Type	Content
1-20	Regular	Inside bounds
21-22	Dominance	Inside bounds, dominant option
23-26	Linear transform	One criterion exactly on boundary
27-28	Linear transform + 10%	One criterion 10% above bound
29-30	Control	Replicates of 9 and 14

Linear consistency

	Linear model consistency			
Gender	No	Yes	All	% consistent
Female	21	18	39	46,2 %
Male	38	49	87	56,3 %
Unknown	1	0	1	100,0 %
sum	60	67	127	

	Linear model consistency			
Dominance wrong	No	Yes	All	% consistent
0	55	65	120	54,2 %
1	3	2	5	40,0 %
2	2	0	2	0,0 %
sum	60	67	127	

Judgments of importance

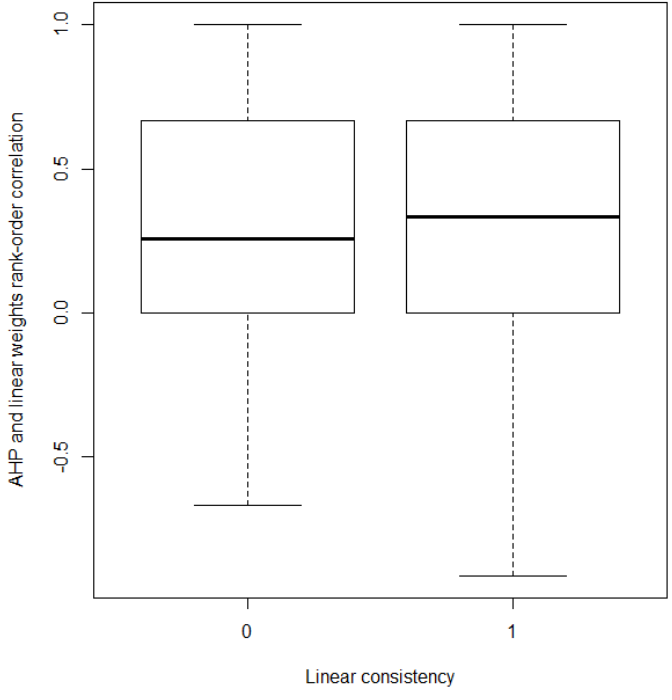
Link of AHP, weights and consistency

AHP-weights correlation

- *Kendall rank correlation*

No difference in consistency

- *Ie. Higher linear consistency is not connected to a larger AHP-weight correlation*
- *Linear consistent subjects are not better in preference consistency*



Link to cognitive style

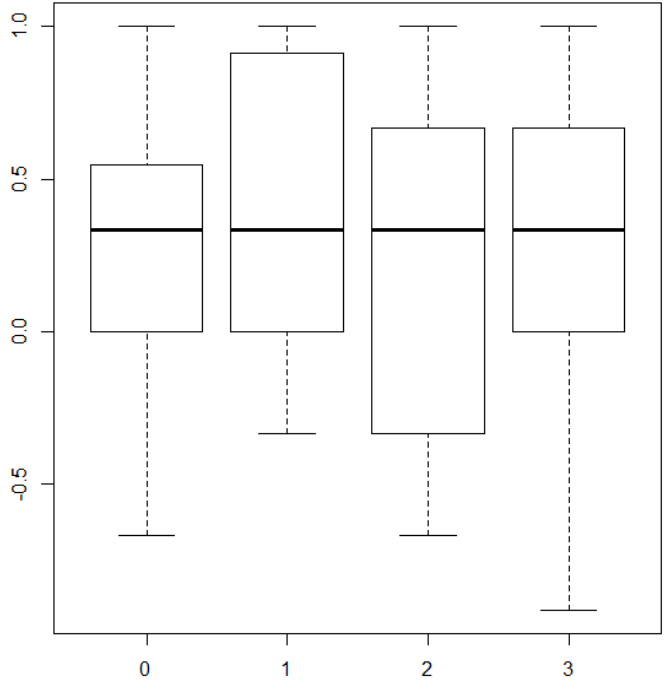
We also checked subject's CRT (Cognitive Reflection Test) score (Frederick, 2005)

CRT-consistency relationship

- *AHP-weights Kendall rank correlation*

No difference

- *Ie. CRT score not related to more consistent preferences*



Prediction

Compared prediction methods

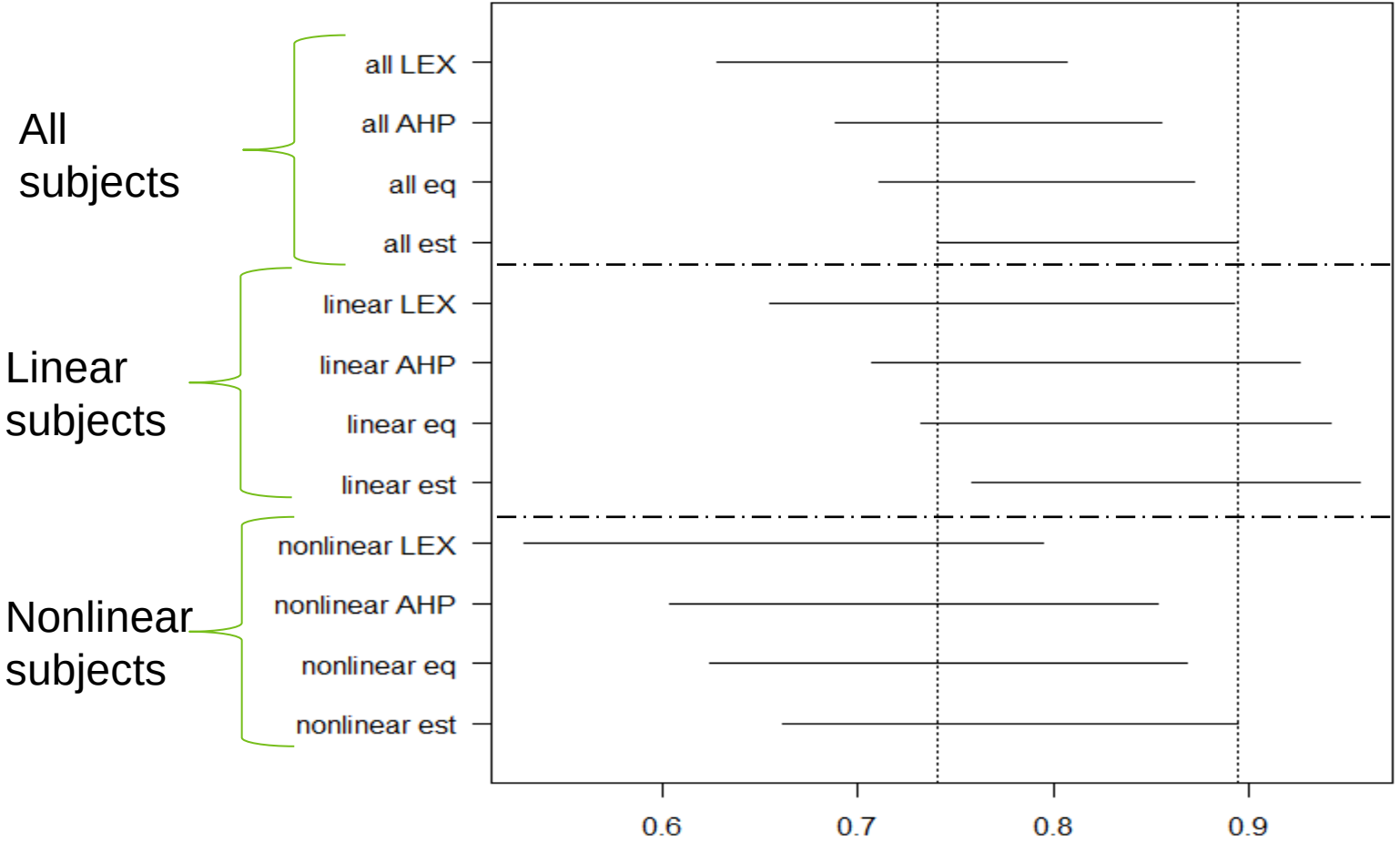
Equal weights (eq)

AHP-based weights (AHP)

Estimated weights (est)

Lexicographic model (LEX)

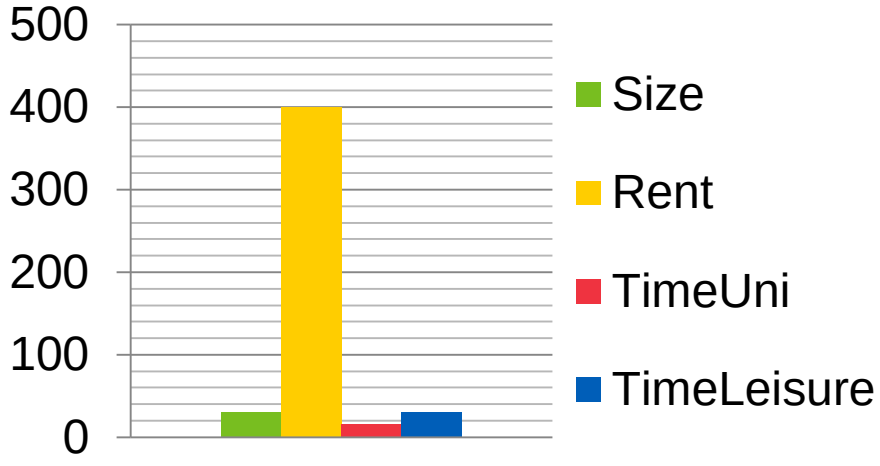
ML estimates + 95% CI, nominal



Scaling

Criteria have very different ranges

- *Size: 20-40 m^2*
- *Rent: 300-500 €/month*
- *Travel time to Uni: 1-30 minutes*
- *Travel time to leisure: 1-60 minutes*

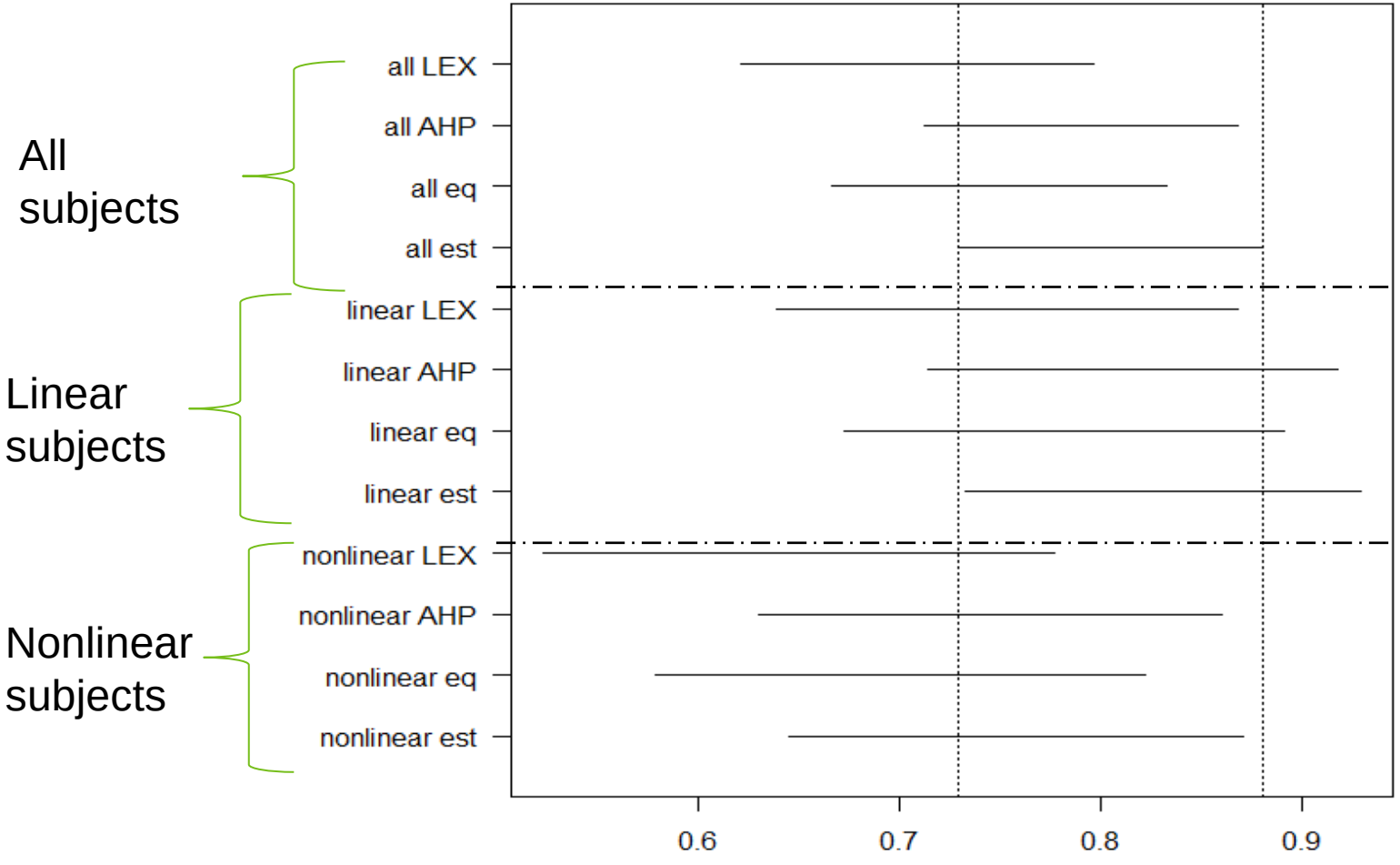


What scales are the subjects using internally?

We tried two options:

- Nominal
- 0-1 normalized

ML estimates + 95% CI, scaled



Methods comparison summary

Nominal	Estimated	Equal	AHP	LEX
Estimated		*No difference*	*Estimated*	Estimated
Equal			*Equal*	*Equal*
AHP				*AHP*
LEX				

Scaled	Estimated	Equal	AHP	LEX
Estimated		*Estimated*	*No difference*	Estimated
Equal			*AHP*	*Equal*
AHP				*AHP*
LEX				

Conclusions

The epsilon formulation is quite suitable for prediction

- Hit rate ~80%
- Caveat: still needed 10 choices as training set

Scaling affects prediction results

- Equal weights lose out with scaling
  - *Argument against Dawes' "choose variables and just add"?*
- Judgments of importance are not informative without scaling
- But, how can we know the internal scale of the subject?

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Questions?